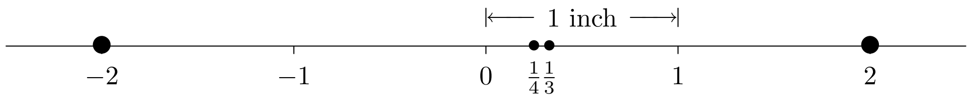
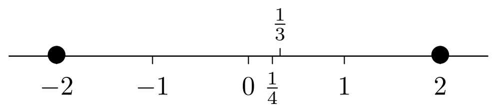
SOLUTIONS TO EXERCISES: THE REAL NUMBERS

IN-SECTION EXERCISES:

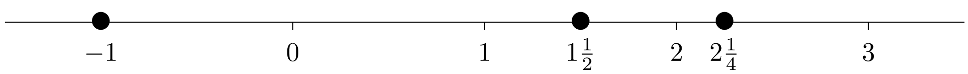
1.



2.



3. Measure the distance from 2 to 3; lay off this same distance to the left of 2 to locate 1; repeat to locate 0.



4. The *nonpositive* real numbers are zero (which is not positive), together with the negative numbers (which are not positive). Be sure to put a solid dot at zero.

5. There are some important subcollections of *the real numbers* that are ... Or, you could read it as: There are some important subcollections of ‘arr’ that are ...

6. $7\,,\ 8\,,\ 9\,,\ 10$ or $7\,,\ 6\,,\ 5\,,\ 4$

7a. the positive whole numbers are $1\,,\ 2\,,\ 3\,,\ \ldots$

7b. Zero is a whole number that is not positive.

7c. All whole numbers are nonnegative.

8. small positive number: $\frac{1}{10{,}000}$ (one ten-thousandth)

large positive number: $2,000,000$ (two million)

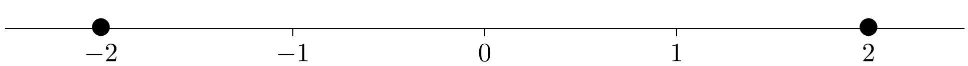
small negative number: $-\frac{1}{10{,}000}$ (negative one ten-thousandth)

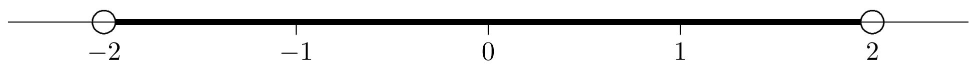
big negative number: $-2,000,000$ (negative two million)

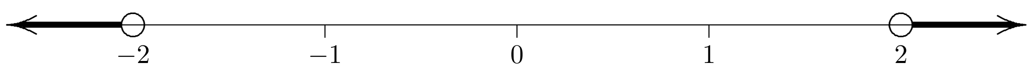
9. The nonnegative numbers are zero, together with the positive real numbers. It doesn’t make sense to say something like ‘the distance from my home to the store is negative two miles’. When you report a distance, you always report a number that is either positive or zero. Thus, size is a nonnegative quantity.

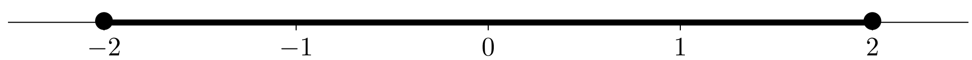
10. $2\,,\ 2\,,\ 0\,,\ 10.2\,,\ 10.2$

11.



12. 

13. 

14. 

15. There is no largest whole number. Given any whole number, you can always get one that is bigger, say by adding 1 .

16. The opposite of $\frac{1}{2}$ is $-\frac{1}{2}$. The opposite of $-\frac{1}{2}$ is $\frac{1}{2}$.

17. I could be thinking of 2 or -2 .

18. I could be thinking of 3 or -3 .

19. -1 is an integer, but not a whole number

20. positive integers: 1 , 2 , 3 , ... negative integers: -1 , -2 , -3 , ...

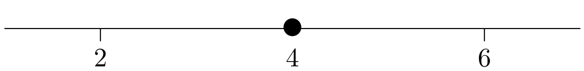
21. nonnegative integers: 0 , 1 , 2 , 3 , ... nonpositive integers: 0 , -1 , -2 , -3 , ...

22. large negative integer: $-10,000$ (negative ten thousand) large positive integer: $10,000$ (ten thousand)

23a. Yes. Since the integers are formed by taking the whole numbers, and throwing in their opposites, we're guaranteed that opposites are in there.

23b. No. Here's how a mathematician would argue this: Call the original (non-integer) x . If its opposite, $-x$, IS an integer, then (from part (a)), the opposite of $-x$, which is x , would also have to be an integer; but it isn't. We've reached a contradiction, so $-x$ must NOT be an integer. (Whew!)

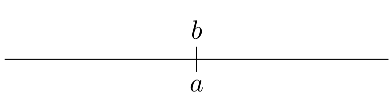
24a. The average of 2 and 6 is $\dfrac{2+6}{2} = \dfrac{8}{2} = 4$.

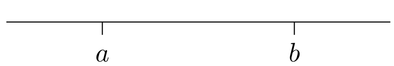
24b. 

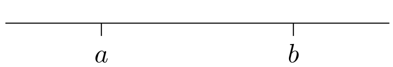
The answers to both (a) and (b) are the same, since the average of two numbers gives the number that is exactly halfway between.

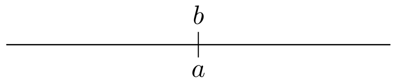
25. The average of 0.13 and 0.14 is $\dfrac{0.13+0.14}{2} = \dfrac{0.27}{2} = 0.135$. The number 0.135 is exactly halfway between 0.13 and 0.14 .

26. If you average two numbers that are the same, then you get the same number. That is, $\dfrac{x+x}{2} = \dfrac{2x}{2} = x$.

27a. 

27b. 

27c. 

27d. 

END-OF-SECTION EXERCISES:

28. SEN, T
29. SEN, ST/SF
30. SEN, T
31. SEN, ST/SF
32. EXP (number)
33. SEN, T
34. SEN, F   (-1 is an integer, but is not a whole number)
35. 

(Remember: you want the *real numbers* between -1 and 2, not just the integers.)