SOLUTIONS TO EXERCISES:
NUMBERS HAVE LOTS OF DIFFERENT NAMES!

IN-SECTION EXERCISES:

1. STEP 1: $\overbrace{0}^{\text{0 pets}} + 2 = 2$; write down the number 2, and circle it.

Since you own fewer than 2 pets, go to STEP 2.

STEP 2: $2 - \overbrace{0}^{\text{0 pets}} = 2$; $2 \cdot 2 = \boxed{4}$; write down the number $\boxed{4}$, and put a box around it. Go to STEP 4.

STEP 4: $\overbrace{0}^{\text{0 pets}} \cdot \overbrace{0}^{\text{0 pets}} = 0$; $0 + \boxed{4} = 4$

2. STEP 1: $\overbrace{7}^{\text{7 pets}} + 2 = 9$; write down the number 9, and circle it.

Since you own more than 2 pets, go to STEP 3.

STEP 3: $\overbrace{7}^{\text{7 pets}} - 2 = 5$; opposite is -5 ; $(-5) \cdot 9 = \boxed{-45}$. Write down the number $\boxed{-45}$ and put a box around it.

STEP 4: $\overbrace{7}^{\text{7 pets}} \cdot \overbrace{7}^{\text{7 pets}} = 49$; $49 + (\,\boxed{-45}\,) = 4$

3. $2 \cdot 8 + 5 \cdot 4$: give 2 pieces of candy to each of 8 kids, and 5 pieces of candy to each of 4 kids; OR
give 2 pieces to each of 8 kids, and 4 pieces to each of 5 kids; OR
give 8 pieces to each of 2 kids, and 5 pieces to each of 4 kids.

$5 \cdot 7 + 1$: give 5 pieces of candy to each of 7 kids, with 1 piece left over.

4.

name for 60	information revealed by name
$6 \cdot 10$	give 6 pieces of candy to each of 10 kids; or give 10 pieces of candy to each of 6 kids
$3 \cdot 20$ or $20 \cdot 3$	60 pieces of candy can be evenly distributed among 3 kids, by giving 20 pieces to each
$7 \cdot 8 + 4$ or $8 \cdot 7 + 4$	give 7 pieces of candy to each of 8 kids, with 4 pieces left over
$16 \cdot 3 + 2 \cdot 6$	give 16 pieces to each of 3 kids, and 2 pieces to each of 6 kids; OR give 3 pieces to each of 16 kids, and 6 pieces to each of 2 kids; OR give 16 pieces to each of 3 kids, and 6 pieces to each of 2 kids; OR give 3 pieces to each of 16 kids, and 2 pieces to each of 6 kids.
$\frac{1}{3}(180)$	give one-third of a piece to each of 180 kids

5. The universal set for x is $\mathbb{R}$ because the theorem says 'For all real numbers x …' .

6. You can add zero to any real number, and this doesn't change the identity of the number. Adding zero gives a new *name* for a number, but doesn't change where it *lives* on a real number line. Consequently, the number 0 is often given the fancy name 'additive identity'.

7. The universal set for x is $\mathbb{R}$ because the theorem says 'For all real numbers x …' .

8. You can multiply any real number by 1, and this doesn't change the identity of the number.

Multiplying by 1 gives a new *name* for a number, but doesn't change where it *lives* on a real number line. Consequently, the number 1 is often given the fancy name 'multiplicative identity'.

9. (a) $0 = 5 + (-5) = (-5) + 5 = 5 - 5$

(b) $0 = \frac{1}{2} + (-\frac{1}{2}) = (-\frac{1}{2}) + \frac{1}{2} = \frac{1}{2} - \frac{1}{2}$

(c) $0 = 3.2 + (-3.2) = (-3.2) + 3.2 = 3.2 - 3.2$

(d) $0 = (-7) + 7 = 7 + (-7)$

10. (a) $1 = \frac{5}{5} = 5 \cdot \frac{1}{5} = \frac{1}{5} \cdot 5$

(b) $1 = \frac{1/2}{1/2} = \frac{1}{2} \cdot 2 = 2 \cdot \frac{1}{2}$

(c) $1 = \frac{3.2}{3.2} = 3.2 \cdot \frac{1}{3.2} = \frac{1}{3.2} \cdot 3.2$

(d) $1 = \frac{-7}{-7} = -7 \cdot \frac{1}{-7} = \frac{1}{-7} \cdot (-7)$ When a negative number comes after a centered dot, it is customary to put the negative number insides parentheses, because $\frac{1}{-7} \cdot -7$ can look somewhat confusing.

11. (a) true

(b) Since '$2+3 = 5+1$' is false, the entire sentence '$2+3 = 1+5 = 6$' is false. Students sometimes 'string' things together with equal signs as they work through a calculation, using '=' to mean something like 'I'm going on to the next step'. DON'T DO THIS! BE CAREFUL!

(c) true: $1 + 2 + 3 = 1 + 5$ is true, and $1 + 5 = 6$ is true.

(d) true for all nonzero real numbers t; not defined if $t = 0$

(e) true

(f) $1 + (2 + 3) + 4 = 5$ is false; $5 = 1 + 5$ is false; $1 + 5 = 6 + 4$ is false; $6 + 4 = 10$ is true. The entire sentence is FALSE because there is at least one 'piece' that is false. (Indeed, in this case, three of the four subsentences are false!)

12. $a = b = c = d$ is shorthand for: $a = b$ and $b = c$ and $c = d$

13. (a) $1 = \frac{1 \text{ pint}}{2 \text{ cups}} = \frac{2 \text{ cups}}{1 \text{ pint}}$

(b) $1 = \frac{1 \text{ m}}{100 \text{ cm}} = \frac{100 \text{ cm}}{1 \text{ m}}$

(c) $1 = \frac{1 \text{ bleep}}{3.4 \text{ blop}} = \frac{3.4 \text{ blop}}{1 \text{ bleep}}$

(d) $1 = \frac{10 \text{ kilometers}}{6.21 \text{ miles}} = \frac{6.21 \text{ miles}}{10 \text{ kilometers}}$

14. $\frac{14 \text{ ft}}{0.5 \text{ sec}} = \frac{14 \text{ ft}}{0.5 \text{ sec}} \cdot \frac{1 \text{ mile}}{5280 \text{ ft}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} = \frac{14 \cdot 60 \cdot 60}{0.5 \cdot 5280} \frac{\text{miles}}{\text{hr}} \approx 19.1$ mph: the passing car is going about 19 miles/hour faster than my car.

END-OF-SECTION EXERCISES:

15. EXP (simplest name: 36)

16. SEN, true

17. SEN, false (Don't use '=' to mean that you're going on to the next step!)

18. SEN, always true

19. SEN, true

20. SEN, true

21. EXP (simplest name: 4 yd)

22. $1 = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 4 \cdot \frac{1}{4}$; put in four $\frac{1}{4}$-cup measures!

23. (a) $630 \text{ sec} = 630 \text{ sec} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = \frac{630}{60} \text{ min} = 10.5 \text{ min}$

(b) $525 \text{ sec} = 525 \text{ sec} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} = \frac{525}{60 \cdot 60} \text{ hr} \approx 0.15 \text{ hr}$

(c) $20 \text{ ft/sec} = \frac{20 \text{ ft}}{1 \text{ sec}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{1 \text{ mile}}{5280 \text{ ft}} = \frac{20 \cdot 60}{5280} \text{ miles/min} \approx 0.23 \text{ miles/min}$

(d) $\frac{20 \text{ ft}}{0.5 \text{ sec}} = \frac{20 \text{ ft}}{0.5 \text{ sec}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{1 \text{ mile}}{5280 \text{ ft}} = \frac{20 \cdot 60 \cdot 60}{0.5 \cdot 5280} \text{ miles/hour} \approx 27.3 \text{ miles/hour}$